AS and A level Further Mathematics Practice Paper – Contingency tables and Goodness of fit – Mark scheme

Question	Scheme	Marks
1		

Cholesterol Level	High	Low	
High	7.6	12.4	20
Low	30.4	49.6	80
	38	62	100

M1A1

(2)

H_0 : Cholesterol level is independent of intake of saturated fats(no association)

H_1 : Cholesterol level is not independent of intake of saturated fats (association)

B1

(1)

O	E	$\frac{(O-E)^2}{E}$	$\frac{O^2}{E}$
12	7.6	2.547… or $\frac{242}{95}$	18.947… or $\frac{360}{19}$
8	12.4	1.56129… or $\frac{242}{155}$	5.161… or $\frac{160}{31}$
26	30.4	0.6368… or $\frac{121}{190}$	22.236… or $\frac{845}{38}$
54	49.6	0.3903… or $\frac{121}{310}$	58.790… or $\frac{3645}{62}$

dM1

A1

$\sum \frac{(O-E)^2}{E}$=5.1358234.. <u>or</u> $\frac{1.2^2}{7.6}+\frac{8^2}{12.4}+\frac{26^2}{30.4}+\frac{54^2}{49.6}-100=5.14$

(awrt **5.14**)

A1

(3)

$\nu=(2-1)(2-1)=1$

B1

$\chi^2_1(0.05)=3.841$

B1

(2)

5.14 > 3.841 so sufficient evidence to reject H_0 [Condone "accept H_1"]

M1

Association between cholesterol level and saturated fat intake

A1

(2)

		(10 marks)

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<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>2</td><td>
<table>
<tr><td colspan="2"></td><td>Happiness</td><td></td><td></td></tr>
<tr><td colspan="2"></td><td>Not happy</td><td>Fairly happy</td><td>Very happy</td></tr>
<tr><td rowspan="2">Gender</td><td>Female</td><td>13.51</td><td>41.77</td><td>30.71</td></tr>
<tr><td>Male</td><td>8.49</td><td>26.23</td><td>19.29</td></tr>
</table>
</td><td>M1
A1</td></tr>
<tr><td></td><td>H_0 : Happiness and gender are independent/ not associated
H_1 : Happiness and gender are not independent/ associated</td><td>B1</td></tr>
<tr><td></td><td>
<table>
<tr><td>O</td><td>E</td><td>$\frac{(O-E)^2}{E}$</td><td>$\frac{O^2}{E}$</td></tr>
<tr><td>9</td><td>13.51</td><td>1.508</td><td>5.996</td></tr>
<tr><td>43</td><td>41.77</td><td>0.0361</td><td>44.264</td></tr>
<tr><td>34</td><td>30.71</td><td>0.351</td><td>37.637</td></tr>
<tr><td>13</td><td>8.49</td><td>2.402</td><td>19.915</td></tr>
<tr><td>25</td><td>26.23</td><td>0.0575</td><td>23.829</td></tr>
<tr><td>16</td><td>19.29</td><td>0.560</td><td>13.274</td></tr>
</table>
</td><td>dM1
A1</td></tr>
<tr><td></td><td>$\sum \frac{(O-E)^2}{E} = 4.91$ or $\sum \frac{O^2}{E} - N = 144.91 - 140 = 4.91$</td><td>A1</td></tr>
<tr><td></td><td>$\nu = (3-2)(2-1) = 2$</td><td>B1</td></tr>
<tr><td></td><td>$\sum \frac{(O-E)^2}{E} < 5.991$</td><td>B1ft</td></tr>
<tr><td></td><td>4.91 < 5.991 so ‘insufficient evidence to reject H0 ’ or ‘Accept H0’</td><td>M1</td></tr>
<tr><td></td><td>No association between gender and happiness.</td><td>A1</td></tr>
<tr><td></td><td></td><td>(10 marks)</td></tr>
</table>

AS and A level Further Mathematics Practice Paper – Contingency tables and Goodness of fit – Mark scheme

Question	Scheme	Marks

3

Defect Type / Shift	D_1	D_2	
First Shift	47.25	15.75	63
Second Shift	56.25	18.75	75
Third Shift	46.5	15.5	62
	150	50	200

M1A1

H_0 : Type of defect is independent of Shift (no association)

H_1 : Type of defect is not independent of Shift (association)

B1

O	E	$\frac{(O-E)^2}{E}$	$\frac{O_i^2}{E_i}$
45	47.25	0.1071…	42.857...
18	15.75	0.3214…	20.571..
55	56.25	0.02777…	53.777...
20	18.75	0.0833…	21.333...
50	46.5	0.2634…	53.763...
12	15.5	0.7903…	9.290...

M1A1

$\frac{(O-E)^2}{E}$=1.5934.. or $\frac{O_i^2}{E_i}$-200=201.5934-200=1.5934..

awrt1.59

A1

$\nu = (3-1)(2-1) = 2$

B1

$\chi^2_2(0.10) = 4.605$

B1ft

1.59<4.605 so insufficient evidence to reject H_0

M1

Insufficient evidence to support manager's belief /claim.

A1

(10 marks)

AS and A level Further Mathematics Practice Paper – Contingency tables and Goodness of fit – Mark scheme

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th></tr>
<tr><td rowspan="2">4(a)</td><td>$\frac{72\times50}{150}=24,\ \frac{78\times50}{150}=26$</td><td>M1</td></tr>
<tr><td>$\frac{72\times64}{150}=30.72,\ \frac{78\times64}{150}=33.28$</td><td>A1
(2)</td></tr>
<tr><td>4(b)</td><td>H_0 :Perceived (body) weight is independent of gender (no association)
H_1 :Perceived (body) weight is not independent of gender (association)
<table><tr><th>O</th><th>E</th><th>$\frac{(O-E)^2}{E}$</th><th>$\frac{O^2}{E}$</th></tr><tr><td>20</td><td>17.28</td><td>0.428148</td><td>23.14815</td></tr><tr><td>22</td><td>24</td><td>0.166667</td><td>20.16667</td></tr><tr><td>30</td><td>30.72</td><td>0.016875</td><td>29.29688</td></tr><tr><td>16</td><td>18.72</td><td>0.395214</td><td>13.67521</td></tr><tr><td>28</td><td>26</td><td>0.153846</td><td>30.15385</td></tr><tr><td>34</td><td>33.28</td><td>0.015577</td><td>34.73558</td></tr><tr><td>150</td><td>150</td><td>1.176327</td><td>151.1763</td></tr></table>$\sum\frac{(O-E)^2}{E}$ or $\sum\frac{O^2}{E}-150=1.18$
$\nu=(3-1)(2-1)=2,\ \chi^2_2(10\%)=4.605$
(Accept H_0) Perceived (body) weight is independent of gender (no association)</td><td>B1
M1A1
A1
B1B1ft
A1ft
(7)</td></tr>
<tr><td>4(c)</td><td><table><tr><th>O</th><th>E</th><th>$\frac{(O-E)^2}{E}$</th><th>$\frac{O^2}{E}$</th></tr><tr><td>36</td><td>50</td><td>3.92</td><td>25.92</td></tr><tr><td>50</td><td>50</td><td>0</td><td>50</td></tr><tr><td>64</td><td>50</td><td>3.92</td><td>81.92</td></tr><tr><td>150</td><td>150</td><td>7.84</td><td>157.84</td></tr></table>$\sum\frac{(O-E)^2}{E}$ or $\sum\frac{O^2}{E}-150=7.84$
$\nu=2,\ \chi^2_2(2.5\%)=7.378$</td><td>B1
M1A1
A1
A1
(5)</td></tr>
<tr><td></td><td></td><td>(14 marks)</td></tr>
</table>

AS and A level Further Mathematics Practice Paper – Contingency tables and Goodness of fit – Mark scheme

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>5(a)</td><td>$\frac{282\times100}{600}$ (Do not accept 282 – 114.2 – 90.2 – 30.6 (o.e.))</td><td>B1
(1)</td></tr>
<tr><td>5(b)</td><td>9</td><td>B1
(1)</td></tr>
<tr><td>5(c)</td><td>2.5 or better (Do not accept 0.025)</td><td>B1
(1)</td></tr>
<tr><td>5(d)</td><td>H_0: hair colour occurs in the ratio 2:6:1:3
H_1: hair colour does not occur in the ratio 2:6:1:3

<table>
<tr><td></td><td>black</td><td>brown</td><td>red</td><td>blonde</td></tr>
<tr><td>observed</td><td>105</td><td>282</td><td>48</td><td>165</td></tr>
<tr><td>expected</td><td>100</td><td>300</td><td>50</td><td>150</td></tr>
<tr><td>$\frac{(O_i-E_i)^2}{E_i}$</td><td>0.25</td><td>1.08</td><td>0.08</td><td>1.5</td></tr>
<tr><td>$\frac{O_i^2}{E_i}$</td><td>110.25</td><td>265.08</td><td>46.08</td><td>181.5</td></tr>
</table>
$\sum \frac{(O_i-E_i)^2}{E_i} = 2.91$ or $\sum \frac{O_I^2}{E_I} - 600 = 602.91 - 600 = 2.91$
(awrt 2.91)
$\nu = 3$
cv is 7.815
[2.91< 7.815] so insufficient evidence to reject H_0 or not significant
There is evidence to suggest that hair colour does occur in the given ratio.</td><td>B1

B1expected
M1
A1

A1

B1
B1
dM1
A1
(9)</td></tr>
<tr><td></td><td></td><td>(12 marks)</td></tr>
</table>

AS and A level Further Mathematics Practice Paper – Contingency tables and Goodness of fit – Mark scheme

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>6(a)</td><td>The seeds are independent / There are a fixed number of seeds in a row / There are only
two outcomes to the seed germinating – either it germinates or it does not / The probability
of a seed germinating is constant</td><td>B1 B1
(2)</td></tr>
<tr><td>6(b)</td><td>$\frac{(0\times2)+(1\times6)+(2\times11)+(3\times19)+(4\times25)+(5\times32)+(6\times16)+(7\times9)}{120\times7}=$
$\frac{504}{840}$
$= 0.6$ **</td><td>M1
A1cso
(2)</td></tr>
<tr><td>6(c)</td><td>$p = 0.6 \quad q = 0.4$
$s = 120\times 21q^5p^2 = 120\times21 \text{ x } 0.4^5 \text{ x } 0.6^2 = 9.29$
$t = 120\times35q^3p^4 = 120\times35 \text{ x } 0.4^3 \text{ x } 0.6^4 = 34.84$</td><td>B1
B1
(2)</td></tr>
<tr><td>6(d)</td><td>H_0 : A binomial distribution is a suitable model.
H_1: A binomial distribution is not a suitable model.
<table><tr><td>Observed number of rows</td><td>19</td><td>19</td><td>25</td><td>32</td><td>25</td></tr><tr><td>Expected number of rows</td><td>11.55</td><td>23.22</td><td>34.84</td><td>31.35</td><td>19.04</td></tr><tr><td>$\frac{(O-E)^2}{E}$</td><td>4.81</td><td>0.77</td><td>2.78</td><td>0.013</td><td>1.87</td></tr><tr><td>$\frac{O^2}{E}$</td><td>31.26</td><td>15.55</td><td>17.94</td><td>32.66</td><td>32.83</td></tr></table>$\nu = 5 - 2 = 3$
Critical value for $\chi^2 = 11.345$
$\sum \frac{(O-E)^2}{E} = 10.23$ or $\sum \frac{O^2}{E} - N = 130.23 - 120 = 10.23$
$10.23 < 11.345$ therefore do not reject H_0
A binomial is a suitable model.</td><td>B1
M1
B1ft
B1ft
M1A1
A1
(7)</td></tr>
<tr><td></td><td></td><td>(13 marks)</td></tr>
</table>

AS and A level Further Mathematics Practice Paper – Contingency tables and Goodness of fit – Mark scheme

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>7(a)</td><td>Mean= $\frac{1\times16+2\times20+...+6\times8}{100}=2.91$ **ag**</td><td>M1A1

(2)</td></tr>
<tr><td>7(b)</td><td>$p=\frac{2.91}{6}=0.485$
$a=100\times C_3^6\times0.485^3\times0.515^3=31.17$
$b=100\times0.485^6=1.3(0)$</td><td>B1
M1A1
A1
(4)</td></tr>
<tr><td>7(c)</td><td>H_0 :Binomial is a good fit
H_1 :Binomial is a not a good fit

<table><tr><td>Number of defective items</td><td>0 or 1</td><td>2</td><td>3</td><td>4</td><td>5 or 6</td></tr><tr><td>O</td><td>22</td><td>20</td><td>23</td><td>17</td><td>18</td></tr><tr><td>E</td><td>12.41</td><td>24.82</td><td>31.17</td><td>22.01</td><td>9.59</td></tr></table>
$\sum\frac{(O-E)^2}{E}=\frac{(22-12.41)^2}{12.41}+\frac{(20-24.82)^2}{24.82}+...+\frac{(18-9.59)^2}{9.59}=18.998...$
awrt 19.0
ν=5-2=3 degrees of freedom
$\chi_3^2(5\%)=7.815$
18.998...>7.815 so reject H_0
Binomial is a not a good fit (and is not a good model for the number of defective items in samples of size 6)</td><td>B1

M1

M1A1

B1
B1ft
M1
A1

(8)</td></tr>
<tr><td></td><td></td><td>(14 marks)</td></tr>
</table>

AS and A level Further Mathematics Practice Paper – Contingency tables and Goodness of fit – Mark scheme

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>8(a)</td><td>
H_0 :Binomial with $p = 0.3$ is a good fit
H_1 :Binomial with $p = 0.3$ is a not a good fit

<table>
<tr><td></td><td>0</td><td>1</td><td>2 or more</td></tr>
<tr><td>Observed</td><td>6</td><td>25</td><td>19</td></tr>
<tr><td>Expected</td><td>50×0.2401
= 12.005 or 12.01
or 12.00</td><td>50×0.4116
= 20.58</td><td>50×0.2646+50×0.0756+50×0.0081
= 13.23+3.78+0.405
= 17.415 or 17.41 or 17.42</td></tr>
<tr><td>$\frac{(O-E)^2}{E}$</td><td>3.003751</td><td>0.949291</td><td>0.144256</td></tr>
<tr><td>$\frac{O^2}{E}$</td><td>2.998751</td><td>30.36929</td><td>20.072926</td></tr>
</table>
$\sum \frac{(O-E)^2}{E} = 4.097...$ or $\sum \frac{O^2}{E} - N = 54.097... - 50 = 4.097...$
awrt 4.09-4.1(0)
$\nu = 3 - 1 = 2$ degrees of freedom
$\chi^2_2(5\%) = 5.991$ (> 4.1(0))
Insufficient evidence to reject H_0 (Accept H_0)
Binomial with $p = 0.3$ is a good fit
</td><td>
B1

M1A1

dM1A1

B1ft
B1ft

A1
(8)
</td></tr>
<tr><td>8(b)</td><td>
$\bar{x} = \frac{40 + 62 + 54 + 24}{100} = 1.8$
$r = 26.78$
$s = 16.07$
</td><td>
B1 cao
B1 cao
B1 cao
(3)
</td></tr>
</table>

AS and A level Further Mathematics Practice Paper – Contingency tables and Goodness of fit – Mark scheme

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>8(c)</td><td>
H_0 :Poisson is a good fit
H_1 : Poisson is a not a good fit
<table>
<tr><td></td><td>0</td><td>1</td><td>2</td><td>3</td><td>4 or more</td></tr>
<tr><td>Observed</td><td>5</td><td>40</td><td>31</td><td>18</td><td>6</td></tr>
<tr><td>Expected</td><td>16.53</td><td>29.75</td><td>26.78</td><td>16.07</td><td>10.87</td></tr>
<tr><td>$\frac{(O-E)^2}{E}$</td><td>$\frac{11.53^2}{16.53}=8.042...$</td><td>$\frac{10.25^2}{29.75}=3.532...$</td><td>$\frac{4.22^2}{26.78}=0.665...$</td><td>$\frac{1.93^2}{16.07}=0.232...$</td><td>$\frac{4.87^2}{10.87}=2.182...$</td></tr>
<tr><td>$\frac{O^2}{E}$</td><td>$\frac{5^2}{16.53}=1.512...$</td><td>$\frac{40^2}{29.75}=53.782...$</td><td>$\frac{31^2}{26.78}=35.885...$</td><td>$\frac{18^2}{16.07}=20.162...$</td><td>$\frac{6^2}{10.87}=3.312...$</td></tr>
</table>
$\sum\frac{(O-E)^2}{E}=14.65-14.56$ or $\sum\frac{O^2}{E}-N=114.65-100=14.65-14.66$
ν = 5 – 1 – 1 = 3 degrees of freedom
$\chi^2_2(1\%)$ = 11.345 (< 14.65)
Sufficient evidence to reject H_0
Poisson is a not a good fit
</td><td>
B1

M1A1
B1 cao
B1ft

A1 cao
(6)
</td></tr>
<tr><td></td><td></td><td>(17 marks)</td></tr>
</table>

AS and A level Further Mathematics Practice Paper – Contingency tables and Goodness of fit – Mark scheme

	Source paper	Question number	New spec references	Question description	New AOs
1	S3 2013	1	FS1 6.1	Continency tables	1.1a, 1.1b, 2.1, 2.5, 3.4, 3.5a
2	S3 2014	3	FS1 6.1	Contingency tables	1.1b, 2.1, 2.2b, 2.5, 3.1b
3	S3 2011	3	FS1 6.1	Contingency tables	1.1b, 2.2b, 2.5, 3.3, 3.4
4	S3 2017	4	FS1 6.1	Contingency tables	1.1b, 2.2b, 2.4, 2.5, 3.1b
5	S3 2013R	4	FS1 6.1	Goodness of fit, Contingency tables	1.1b, 1.2, 2.1,2.2b, 2.5, 3.1b, 3.3, 3.4
6	S3 2014	5	FS1 6.1	Goodness of fit	1.1a, 1.1b, 2.1, 2.2b, 2.5, 3.4, 3.5a
7	S3 2012	6	FS1 6.1	Goodness of fit	1.1b, 2.1, 2.2b, 2.5, 3.4
8	S3 2016	6	FS1 6.1	Goodness of fit	1.1b, 2.2b, 2.5, 3.1b, 3.3, 3.4